A QUAD BGO DETECTOR AND ITS TIMING AND POSITIONING DISCRIMINATION FOR POSITRON COMPUTED TOMOGRAPHY

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Quad BGO detectors and the timing and positioning discriminators have been developed for high sensitivity multilayer positron computed tomographs. Each detector consists for rectangular BGO crystals and two cylindrical photomultiplier tubes. The design allows good optical coupling between the crystals and the photomultiplier tubes, which is essential in order to obtain good time resolution with reasonable spatial resolution of the system. The discriminator consists of a time pick-off circuit based on the first photoelectron detection method and a positioning circuit. The positioning circuit identifies the crystal absorbing an annihilation photon.

The design criteria of the electronic system and the performance are described. With a suitable optical configuration of the detector, the erroneous positioning due to statistical noise is negligibly small and the coincidence time resolution for annihilation photon pairs is about 3.6 ns fwhm. The unit works satisfactorily at a count rate up to at least 360 kcps.

1. Introduction

The development of positron computed tomography and its applications in nuclear medicine have progressed very rapidly in recent years [1]. The technique makes it feasible to visualize sectional images of positron-emitting radio-pharmaceuticals given to a patient. The technique is based on the coincidence detection of annihilation photon pairs, and sectional images can be reconstructed by a computer using a similar principle to that used in X-ray computed tomography. As a result of the capability of quantitative three-dimensional imaging together with the clinical usefulness of cyclotron-produced positron emitters ($^{11}$C, $^{13}$N, $^{15}$O, $^{18}$F, etc.), much effort is still being devoted to the development of systems with higher sensitivity, spatial resolution and counting speed.

A variety of devices have been designed and constructed so far. A typical detector configuration is a single- or multi-layer ring array of scintillation detectors surrounding the patient body. To attain high detection sensitivity with reasonable spatial resolution in this type of detector system, two requirements are essential: one is that a large number of small scintillators should be arranged on the ring with as high a packing ratio as possible, and the other is that the scintillators should have high stopping power for annihilation photons. Rectangular scintillation crystals are more preferable than cylindrical ones for the high packing ratio. The width of the crystals along the ring circumference limits the intrinsic spatial resolution of the obtained image, while the height of the crystal (along the detector ring axis) determines the thickness of the slice to be imaged. The width is usually chosen to be somewhat smaller than the height of the crystal, because the reconstructed images usually suffer from further smoothing in the software in order to suppress statistical noise in the images.

Bismuth germanate (BGO : Bi$_4$Ge$_3$O$_{12}$) is the most suitable scintillation material for this application because of its high gamma-ray stopping power [2], but its relatively low scintillation efficiency [8–16% of NaI(Tl)] and its long scintillation decay time (300 ns) require efficient optical coupling to photomultiplier tubes in order to obtain good time resolution and reasonable energy resolution. One-to-
null coupling between a rectangular BGO crystal and a cylindrical photomultiplier is apparently unsatisfactory for efficient optical coupling in a tightly packed multi-layer system.

A solution of this difficulty may be the use of position-sensitive detector units. An example is the use of a large crystal viewed by two photomultiplier tubes, the position of excitation in the crystal is determined from the ratio of the signal outputs of the two tubes. Ter-Pogossian and his group developed such detectors using NaI (TI) crystals for thin multi-layer systems [2-3]. In this system, 64 detector units are arranged in a circular ring, and position information in each detector is used to separate the layers. However, this principle cannot be used for the present purpose involving the use of BGO crystals, because the poor light yield of BGO will result in insufficient positioning resolution. In addition, if we use this principle to extract position information along the detector ring circumference, misalignment of the positioning electronics may cause appreciable distortion in the images obtained.

In order to overcome these difficulties, we have constructed a quad BGO detector unit, which is composed of four BGO crystals stuck together and two photomultiplier tubes [6,7]. The crystal positioning is performed by applying a principle similar to that employed in Ter-Pogossian's detector, but the position-sensitive configuration provides more accurate positioning and timing properties. This detector was developed for a detector unit of a multi-layer whole-body positron emission tomograph, in which the detectors are arranged along the ring circumference of each detector ring. This paper describes the positioning property of the quad BGO detector, the timing and positioning discriminator, and the overall performance of this detector unit.

2. Principle of the quad BGO detector

The configuration of the quad BGO detector is shown in fig. 1. The BGO crystals (supplied by the Hitachi Chemical Co., Ltd.) are 13 mm wide (H), 30 mm high (L) and 25 mm long (D). Each photomultiplier tube of PMT-X and PMT-Y is an HTV R 1362 (supplied by the Hamamatsu TV Co., Ltd.) having a diameter of 29 mm (D). HTV R 1362 has high quantum efficiency, uniform photomultiplier sensitivity and small tube-to-tube variation in transit time. The two crystals C2 and C3 are optically coupled to each other with transparenc separation (silicon rubber), while the inner crystals C1 and C4 are optically shielded with light reflector (MY580) from the outer crystals. Each of the photomultiplier tubes is coupled to the two crystals with transparent silicon rubber. The other surfaces of the crystals are coated with the light reflector.

A coincidence in one of the outer crystals produces a signal from one of the photomultiplier tubes, while that in the inner crystal yields coincident signals from the two tubes with different amplitudes. The uses of the two signals can be made almost completely with the same crystal absorbs a photon of a given energy. The reflecting crystal is identified from the relative pulse-heights of the two signals because the pulse-height ratio of the two signals largely depends on which of the crystals absorbs the photon. This is a result of the large difference of the refractive indices of the BGO crystal (n = 2.1 at 390 nm wavelength [8]) in the time is much higher than that of the transparent silicon rubber (n = 1.45).

The positioning property is shown more clearly in fig. 2, which is a contour display of the two-dimensional pulse-height distribution of the coincident signals from the two photomultipliers tubes. Coordinates of X and Y represent the pulse-heights of the signals from PMT-X and PMT-Y, respectively. The distribution was obtained by irradiating the four crystals uniformly with a broad beam of annihilation radiation. Note that the distribution has four well separated peaks B1 through B4 even though the positions of the crystals are visualized individually in the crystals.

The peaks B1 through B4 correspond to full energy absorption of annihilation photons in the crystals C1 through C4, respectively. The peaks B1 and B4 lie on the annihilation axis, while B2 and B3

Fig. 1. Schematic drawing of the quad BGO detector.
Fig. 2. Contour display of the two-dimensional pulse-height distribution of the coincident signals from the two photomultiplier tubes. Coordinates X and Y represent the pulse-heights of the signals from PMT-X and PMT-Y, respectively. The distribution was obtained by irradiating the tubes with a broad beam of annihilation radiation.

distribute two-dimensionally. The four peaks are on a straight line (broken line) at -45° with respect to the X-axis. This indicates that the sum of the signals X and Y can be used as an energy signal. The line D2 indicates an energy discrimination level (350 keV), and three lines P1, P2, and P3 represent the partitioning lines of the positioning discriminator described later.

The firm regions divided by the partitioning lines P1, P2, and P3 are characterized by crystal efficiencies for the identification of the crystals absorbing the photons.

Fig. 3a is a similar result obtained by irradiating the detector with a narrow beam, 2 mm wide, at the boundary between the two crystals C1 and C2. Fig. 3b represents the result at the boundary between C2 and C3. Note that the full energy peak of the crystal C2 is located at almost the same position as in both Figs. 3a and b. The fact implied that average pulse-height ratio of the signals X and Y is almost independent of the positions of subminiholes in each crystal.

3. Theoretical analysis of the positioning performance

A good positioning performance of the detector is obtained with a reasonable distribution of photomultiplier tubes and the suitable setting of the partitioning lines of the positioning discriminator. We will consider the optimum condition which minimizes erroneous divisions.
Suppose that the pulse-height ratio of the coincident signals \( X \) and \( Y \) from the two photomultiplier tubes is independent of the position of scintillations in each crystal as described above. Then, we can consider that the solution of the full energy absorption peaks in fig. 2 must arise from two causes. One is the statistical fluctuation of the effective number of photoelectrons accumulated at the first dynodes of each photomultiplier tube. The other is the successive excitation of the two adjacent crystals by a single interaction of a photon.

The statistical fluctuation depends on the average number of total photoelectrons, pulse-shaping in the positioning discriminator and the time when the positioning signal is activated. The fluctuation can be represented by Poisson statistics associated with "the effective photoelectron number". The effective photoelectron number is defined by the reciprocal of the relative statistical variance of the pulse amplitude of the shaped signals in the discriminator at the time of the positioning (see Appendix).

First, we assume that the statistical spread is negligible. Thus, the full energy absorption events in the individual crystals will be positioned at the maximum points of the corresponding peaks. These points are represented by \( M_i \) through \( M_4 \) in fig. 4. The maximum points will be on straight line defined by

\[
y = -x + M \gamma,
\]

where \( \gamma \) is the slope of the effective photoelectron number for the \( X \) and \( Y \) signals, and \( M \) is the gain of the photomultiplier-amplifier system. The coordinate of the four points \( M_i \); through \( M_4 \) are given by \((M, 0), (Y, M), (M, N), (0, N \gamma)\), respectively, where

\[
M_x = f M, \quad M_y = (1 - f) M.
\]

\( f \) is the distribution constant defined by the fraction of activation light received by PMT-X (or PMT-Y) for a scintillation in the crystal C3 (or C2).

The event by multiple interaction of the photon in the two adjacent crystals, however, is positioned at the intermediate point between the maximum points corresponding to the two crystals. We call such an event the cross-talk event. It can be readily shown that the position of the cross-talk event depends on the serial number between the two maximum points at a ratio equal to the average of the scattered energy in the individual crystals. The cross-talk events are mainly the result of photons incident near the boundary between the two crystals. The events can be seen in figs. 3a and b as each middle between the two peaks.

To determine the cross-talk events into the two regions equally, the positioning line must pass through the center between the two maximum points. This is, the positioning line \( P_2 \) must be aligned to \( y = x \), while \( P_1 \) and \( P_3 \) must be aligned to \( y = M_x x \) and \( y = M_y x \), respectively, where

\[
M = (2 - f)/f.
\]

Second, the statistical spread of the peaks will be considered neglecting the cross-talk events. If we assume that there is no correlation between the statistical fluctuation of the two coincident signals, the distribution of the full energy absorption peak for the crystal C3, for example, is represented by a two-dimensional Gaussian function \( p(x, y) \):

\[
p(x, y) = \frac{1}{2\pi \sigma_x \sigma_y} \exp \left\{ -\frac{1}{2} \left[ \frac{(x - M_x)^2}{\sigma_x^2} + \frac{(y - M_y)^2}{\sigma_y^2} \right] \right\},
\]

where

\[
\sigma_x^2 = A((1 - f)/f)^{1/2}, \quad \sigma_y^2 = A((1 - f)/f)^{1/2}.
\]

Events positioned in the region beyond the positioning line \( P_2 \) and \( P_3 \) cause erroneous addresses from the other crystal signals. We will introduce two functions \( C_1 \) and \( C_2 \) of the wrong addresses which are defined by the mathematical planes of the events in the regions of \( y > x \) and \( y < x \), respectively. The wrong
addressing functions $E_1$ and $E_2$ are given by

$$E_1 = \frac{1}{(2\pi)^{1/2} \beta_1} \int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{2\beta_1^2}\right) \, dx,$$

$$E_2 = \frac{1}{(2\pi)^{1/2} \beta_2} \int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{2\beta_2^2}\right) \, dx,$$

where

$$\beta_1 = \frac{m f - M_2}{(1 + m^2)^{1/2}}, \quad \beta_2 = \left(\frac{m^2 e_a^2 + e_b^2}{1 + m^2}\right)^{1/2},$$

$$U_x = -\frac{M_2 + M_2}{1 + m^2}, \quad U_y = \frac{M_2 + M_2}{2}.$$

Using eqs. (2), (3) and (5), we have

$$E_1 = \phi\left[-\left(\frac{dV}{d} - cy\right)^{1/2}\right],$$

$$E_2 = \phi\left[(1 - 2y)\lambda^{1/2}\right],$$

where $\phi(x)$ is the Gaussian error function:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{t^2}{2}\right) \, dt.$$

We define the optimum distribution coefficient $f_0$ which allows the larger of $E_1$ and $E_2$ to be a minimum. Considering that $E_1$ and $E_2$ are monotonically decreasing and increasing functions of $f (0 < f < 1)$, respectively, we can show that the optimum distribution coefficient is

$$f = f_0 = 1/2.$$

Under this condition, $f = f_0$, we have

$$E_1 = E_2 = \phi(-\lambda^{1/2}),$$

$$\beta_1 = \frac{\lambda}{\lambda^{1/2}} - 1/2, \quad \beta_2 = \frac{\lambda e_a}{\lambda^{1/2}}.$$

Eq. (11) implies that the segment M_2 is divided into three equal parts by the points M_1 and M_2. We call this situation the optimum configuration. Eq. (12) gives lines of $y = 0.2x$ and $y = 0.5x$, which we call the optimum partitioning lines of $F_1$ and $F_2$.

The distribution coefficient depends on the light detector, the size of the crystals, the orientation of the crystal and the transparent adhesive, and so on. The present detector has almost the optimum configuration shown in Fig. 2. The derivative of the peak pulse from the optimum cell has time 3% of the distance between the maximum points of the outer peaks $M_1$ and $M_2$. The distribution coefficient is estimated to be about 0.31, that is $|f - f_0| < 0.02$.

6. Design principles for the discriminator

The block diagram in Fig. 5 represents design principles of the timing and partitioning discriminator for the quad BGO detector. The discriminator is composed of two circuits. One is a time pick-off circuit and the other is a partitioning discriminator.

The mode signals are summed with direct coupling of the two photomultiplier tube and fed to a leading-edge discriminator. It provides timing pulses with detection of the first photoelectrons in the upper anode as the timing discriminator reported previously [9].

On the other hand, an energy signal is obtained by summing up the dynode signals $X$ and $Y$ with an analog preamplifier $(X + Y)$. The energy signal is fed to a pulse-height discriminator, the output of which is used for gating the corresponding timing pulse. The pulse-positioning pulses $P_1$ and $P_2$ are extracted from two pick-off circuits, each of which is composed of a simple sequencer and a sequencer. One is $(X - Y)$ and $P_1$, and the other is $(X - Y) + (X + Y)$ and $P_2$, where $\text{Max}(X, Y)$ is the larger of $X$ and $Y$, $c$ is a variable constant, and $P_1$ and $P_2$ are comparators.

The second circuit corresponds to the partitioning line $P_0$ of $y = x$ in Fig. 2, while the latter corresponds to both $P_1$ and $P_2$. The position of the line $P_0$ is represented in terms of 8 in following:

$$P_0 = \frac{1}{2} x - 1.$$

If $k$ is adjusted to be $1/2$, the lines $P_0$, $P_1$ and $P_2$ are aligned to the optimum partitioning lines of $y = 0.2x$ and $y = 0.5x$. The block diagram of the timing and partitioning discriminator is shown in Fig. 5.
5 x, respectively. The optimum positioning lines are shown in Fig. 2.

Photographs in Figs. 6a and 6b are pulse waveforms of the output signals from the counting comparator \( C' \) and \( C'' \) and \([X', \ Y'] = (X + \ Y)\), respectively. The pulses in Figs. 6a-6c are apparently classified into four groups. These four groups from top to bottom correspond to scintillations in the individual crystals from C1 through C4, respectively. The positive and negative pulses in Fig. 6b correspond to scintillations in the outer crystals of C3 and C4, and the inner crystals of C2 and C1, respectively. The summation \( s_{1} \) and \( s_{2} \) divide these pulses according to their polarity.

The binary positioning pulses are derived from the output pulses from \( s_{1} \) and \( s_{2} \) by cleaning with the start pulses, as shown in Fig. 6c. A pair of the binary positioning pulses constitutes with each timing pulse gives one of the time expanded addresses.

5. Circuit description

Fig. 7 shows a circuit diagram of the timing and positioning discrimination. The circuit was constructed to be simple and compact because it has to be provided for each detector unit and the units are closely packed on a circular geometry of a positron computed tomograph.

Current pulses from the anodes of the two photomultiplier tubes are summed and shaped with an RC integrator (time constant 50 ns). The integrated pulse is amplified with a wide-band differential video amplifier A-1 (MC1733 with 200 MHz bandwidth and 1500:1 gain). The capacitive feedback between pins 3 and 16 of MC1733 acts as an additional RC differentiator (time constant 150 ns). The amplified pulse is bipolar with the average non-zero value of 250 mV and is fed to a 1-bit edge-discriminator (zero) which is a fast ECL integrator (half of MC687).

Two delay circuits are provided so that the energy gain pulse arrives at the gate G-2 just align the arrival of the corresponding timing pulse. One is a variable time delay which consists of an RC integrator and an ECL gate G-1. This can be used for time-slicing windows for detector-to-detector variation in a range of 50 ± 10 ns. The other is a 150 ns constant delay line (15 on the time scale 250 ms bandwidth) which is contained in a dual-in-line package. The timing pulses going through the gate G-2 is fed to a multivibrator, which consists of two ECL gates G-3 and E-4. The correct pulse of the multivibrator is a pulse of 30 ns followed by a rectangular dead time of 0.200 ms which is effective in eliminating pulse pile-up error.

On the other hand, the last dynode current pulse of each photomultiplier tube is sharpened to a single pulse by a delay-line clipping circuit (clipping time 150 ns) and is fed to an RC integrator (time constant 220 ns). The sharpened pulses allow a good positioning performance at a high count rate. These pulses are amplified by each of the differential amplifiers A-2 and A-3 (two MC1733's with a differential gain of 100), which have an input dynamic range of 3 V with a feedback loop stabilizing a bias voltage.

The output signals from A-2 and A-3 correspond to \( X \) and \( Y \) in Fig. 5, respectively. The pulse of the signals \( X \) and \( Y \) is applied to the following four circuits:

1) A pair of resistors \( R_{1} \) corresponding to \( (X + \ Y) \) in Fig. 5.

2) A number of resistors \( R_{1} \) corresponding to another \( (X \ Y) \) which is involved in \( R_{1}\text{Max}(X, Y) - (X + Y)\).
Fig. 7. Circuit diagram of the timing and positioning discriminator.
(3) a dual number (μPA38A) corresponding to \( \text{Min}(X, Y) \).

(4) a comparator D-4 (a half of Am687) corresponding to both the circuits of \((X - Y)\) and \(I_1\).

A pulse-height correction circuit is provided for the case, in which the average pulse-height of the summed signals for the outer crystals C2 and C3 is different from that for the inner crystals C1 and C4. Such a case may be caused by the inequality of the light yield of the crystals, the inhomogeneity of the photocathode sensitivity of the photomultiplier tubes, and so on. A differential amplifier A-5 (lndC1733) with a differential gain of 10 and a variable resistor ENERGY ADD. allow the pulse-height correction to be made in the following manner.

The output signal \( Q \) from A-5 is given by

\[
Q = G[pX + (1 - p)Y] \quad \text{if } X > Y,
\]

\[
Q = G[pY + (1 - p)X] \quad \text{if } Y > X,
\]

where \( p \) is a potentiometer (0 <= p <= 1) adjustible with ENERGY ADD. \( X \) is a constant of about 0.5 and \( G \) a signal gain of about 10. The signal \( Q \) for the outer crystals is variable with the parameter \( p \), while \( Q \) for the inner crystals is not so. Thus, variable adjustment of \( p \) reinforces both the average pulse-heights of the summed signals to be equal.

A variable resistor of POSITION ADD. is provided for the adjustment of the parameter \( p \) in eq. (6). A differential amplifier A-5 (lndC1733) with a differential gain of 10 and a comparator D-3 (a half of Am687) act as the timing comparator \( [\text{threshold } X, Y] - \{(X + Y)\} \) and \( I_2 \) in fig. 3, respectively.

A pulse-height discriminator D-2 (a half of Am687) provides gate pulses for a gate G-2 and both the comparators D-3 and D-4. The comparators are unbalanced during the presence of the gate pulses. Gates G-5 and G-6 allow the positioning pulses to be shaped with the corresponding timing pulses. RC integrators at input stage of A-5 and D-4 are provided for shaping the input pulse and for the positioning pulses as extracted from the gate when the shaped pulses reach the zero point (see fig. 6).

6. Experimental results

The performance of the quad BGO detector with the timing and positioning discriminators for annihilation photons was studied.

Fig. 8 shows pulse-height spectra for the individual crystals. Each spectrum was obtained by differentiating a corresponding integral spectrum which was obtained by counting the positioning pulses as a function of the energy discrimination level.

A coincidence gate spectra for annihilation photon pairs with the quad BGO detector and the KEM-100A multichannel discriminator. Each spectrum was obtained by gating the shaping signals with the corresponding positioning signals. The difference between the two spectra of C1 provided the time coincidence of 0.5 ns.
measurement was about 1.8% (FWHM) when the full charge integration was adopted. The following experimental data were obtained with the energy discrimination level set at 350 keV (the minimum point of the valley of the pulse-height spectrum).

Fig. 10 shows coincidence time spectra for annihilation photon pairs obtained with the detector and a time reference detector. The reference detector was a plastic scintillator NPL02A (40 mm diameter x 30 cm) coupled to a 36 mm diameter photomultiplier tube (HTV H 329). The time spectrum for each crystal has the same time resolution of 2.6 ns FWHM, but is shifted in vertex from C1 to C4. The time difference between C1 and C4 was 12.3 ns, which corresponds to the distance between detectors.

X and Y signals are negligibly small as compared with the outputs of 2.6 ns. Fig. 10 shows coincidence time spectra of the four quad BGO detectors. The time resolution was 3.1 ns for C1 and 7.1 ns for C4, with full width at tenth maximum (FWTM) of 80 ns.

Fig. 11 shows positioning responses of the quad BGO detectors in a section of a low beam. The responses were obtained by moving a 3 mm collimated beam of annihilation photons impinging perpendicularly to the crystal face. Each source corresponds to the response of each crystal. All of the responses have a fwhm of 15 ns which is equal to the crystal width. Fig. 12 shows positioning responses of the detector at a high count rate of 360 kcps. The responses were obtained by fixing two sources (a Ge-186 and a Cs-137) in front of the detector and moving a 2 mm collimated beam of annihilation photons impinging perpendicularly to the crystal face. The differences in the peak heights were due to the difference in the crystal and scattering efficiency of the two crystal positions.

7. Discussion and conclusions

The time resolution with detection of the first photonelectron is basically proportional to the average number of initial photoelectrons Np [8]. The positioning property, however, depends on the effective photoelectron number N.
The values of $N_0$ and $N$ can be estimated from measurements of energy resolutions with different integration time constants (16). The relative total variance $V_T$ of the pulse-height for a mono-energetic event is given by

$$V_T = V_B + V_C,$$

where $V_B$ is the relative statistical variance and $V_C$ the relative non-statistical variance. The latter is caused by the intrinsic fluctuation of the scintillation crystal, inhomogeneity of the photocathode of the photomultiplier tube, and so on. Since the relative statistical variance is given by the square of the effective photoelectric number, we have

$$V_B = \frac{1}{N_B} + V_C.$$

For the full charge integration, we have another value of the relative total variance $V_{T0}$:

$$V_{T0} = \frac{1}{N_B} + V_C.$$

On the other hand, the effective photoelectric number in pulse-shaping with an RC integration (time constant $T_R$) at time $t$ is given by

$$N = \frac{\int_{0}^{t} f(t) dt}{f(t) dt} \left( \frac{1}{V_B} + \frac{1}{V_C} \right) N_0,$$

where $T$ is the decay time constant of scintillation (see Appendix). Thus, we use estimates $N_0$, $N$, and $V_C$ from eqs. (14), (15) and (16) using the experimental values of $V_T$ and $V_{T0}$.

From the experimental results of the energy resolution of $2.5\%$ with the positioning discriminator and $10\%$ for the full charge integration, we have $V_T = 0.0113$ and $V_{T0} = 0.0093$. Since the positioning discriminator has $T_1 = 220 ps$ and $T_2 = 300 ps$, eq. (16) gives $N = 0.47 N_0$. Then, we obtain

$$N_0 = 210,$$

$$N = 99,$$

$$V_C = 0.0093.$$

In the previous experiment using a single BGO crystal [9], it was shown that the BGO-BGO coincidence time resolution $\Delta T$ (ns) in terms of approximate given by

$$\Delta T = 780 N_0.$$

Eq. (20) with $N_0 = 210$ gives $\Delta T = 3.7$ ns, which is consistent with the present experimental (20 ns).

From the estimated value of $N = 99$, the erroneous addressing due to the statistical noise can be examined. If the detector has the optimum distribution constant at $V = V_0 = 1$, the wrong addressing (resp. constant) is given by $V = 0.0113$ using eq. (10). The larger of $S_1$ and $S_2$ increases with the increase of $V = V_0$, but it is estimated to be less than $1\%$ even for $V = V_0 = 0.06$ using eqs. (17) and (18). Since the detector has $N = V_0 = 0.92$, the wrong addressing is negligible.

The value of $N = 99$ gives the relative statistical variance:

$$V_B = 0.0113.$$

In comparison with eqs. (19) and (31), $V_C$ is much smaller than $V_B$. It corresponds to the fact that the signals $X$ and $Y$ from the two photomultiplier tubes have negligible correlation, which supports the assumption used in the derivation of eq. (4).

The response curves in fig. 10 display a resonable positioning property except that the tails of the peaks coincide with the adjacent peaks. These tails arise from the intrinsic cross-talk effects due to the multiple interaction of photons. The experimental result in fig. 12 suggests that the detector itself has a normal positioning property at the high count rate of 360 kcps. (about 90 kcps per each crystal). The good quantitative performance confirms that the detector units we examine for dynamic studies with position-computed tomographs.

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Appendix: Effective photoelectron number in pulse-shaping with an RC integrator

The average shape of the original scintillation current pulses for photons of a given energy can be described in terms of photoelectron numbers arriving at the first dynode of the photomultiplier tube in unit time, as follows:

\[ T(t) = \frac{N_0}{\tau} e^{-t/\tau} U(t), \quad (A1) \]

where \( \tau \) is the decay time constant of scintillation, \( N_0 \) the average number of total photoelectrons and \( U(t) \) the step function.

When the original pulses are shaped with an RC integrator (time constant \( T_i \)), the average pulse waveform \( V(t) \) is given by

\[ V(t) = \int_0^t R(t) U(t-s) ds, \quad (A2) \]

where \( R(t) \) is the response function of the circuit, given by

\[ R(t) = \frac{1}{T_i} e^{-t/T_i}. \quad (A3) \]

Owing to the statistical fluctuation of the photoelectrons accumulated, the individual pulse-amplitude \( V(t) \) fluctuates from \( V(t) \). The standard deviation \( \Delta V(t) \) is represented by

\[ \Delta V(t)^2 = \int \Delta V(t)(t-x)^2 \; dx. \quad (A4) \]

Therefore, the relative statistical variance of the integrated pulse-amplitude at time \( t \) is

\[ \frac{\Delta V(t)}{\langle V(t) \rangle} = \frac{1}{T_i^2} \left( \frac{1}{2} \frac{(T_i - T_i^2)}{T_i} - e^{-t/T_i} \right) \quad (A5) \]

The reciprocal of the variance gives the effective photoelectron number expressed by eq. (16).

References